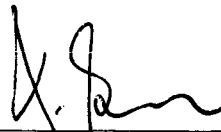NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1164

Small-Sample Evaluation of Squib Time-Current Firing Characteristics

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Abstract

A "universal" firing curve template can be used as a guide for testing hot-wire squibs, provided the squibs exhibit only one significant thermal time constant.

By combining the template with tests by variables, as few as six squibs can provide useful information relating to percentiles in either the all-fire or the no-fire regions.

Small-Sample Evaluation of Squib Time-Current Firing Characteristics

I. Introduction

The firing characteristics of hot-wire squibs, such as that shown in Fig. 1, are commonly presented in the form of current-time curves (Fig. 2). These firing or current-time curves may be prepared by measuring functioning time at various current levels, plotting the results, drawing a line of "best fit" for the 50% line, and supplementing these results with Bruceton, Probit, or other tests to provide an estimate of percentiles at one or more current levels.

Each of these common procedures typically requires destructive testing of several hundred samples, and none of them is well suited to scaling down where only 30 or 40 samples are available.

II. Small Sample Technique

Where sample sizes are severely limited, firing curves may be prepared by an alternate procedure if the squib
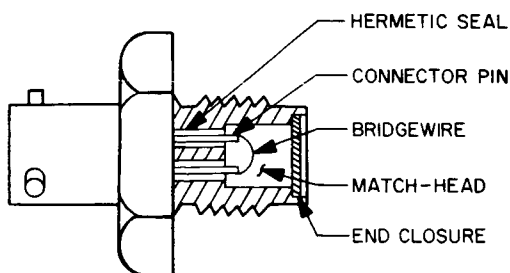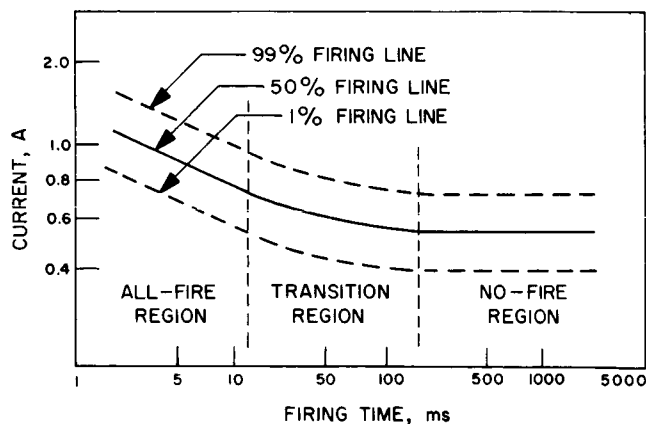

Fig. 1. Typical connector-type squib

Fig. 2. Typical squib firing curves

has only one significant thermal time constant (Refs. 2 and 3).

A. "Universal" Template

A "universal" firing template (Fig. 3) suitable for all squibs having a single significant thermal time constant should be prepared to match the equation

$$\left(\frac{I}{I_N}\right)^2 = \frac{1}{1 - e^{-t/CR}} \qquad (1)$$

where I is the firing current, I_N is the no-fire current, t is the firing time corresponding to I, and CR is the thermal time constant [the derivation of Eq. (1) is given in Appendix A].[1]
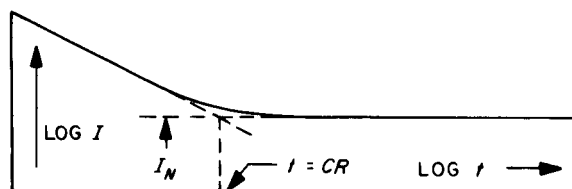

Fig. 3. "Universal" firing template

B. The One-Sample Test[2]

If only one sample were available, the third-harmonic technique (Refs. 2 and 3) could be used to measure the thermal time constant, and the ignition time could be then measured at some reasonably high current level.

These two measurements alone would allow (with obvious reservations) location of the template and drawing of the "first-guess" firing line; the thermal time constant allows the position of the template to be fixed along the time axis, and the time-current point then allows location of the template relative to the current axis.

C. Percentiles in the All-Fire Region

In the all-fire region of Figs. 2 and 3, it may be shown (Appendix A) that Eq. (1) can be expressed in the form:

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[1]The template for such a curve can be prepared conveniently by plotting Eq. (1) on such log-log paper as Keuffel and Esser No. 359-125L, taking I_N as 1 A and CR as 0.01 s.

[2]The concept of a "one-sample" test is introduced for illustrative purposes only; in practice, the measured thermal time constant may differ considerably from that corresponding to the intercept of the all-fire and no-fire lines.

$$I^2 t = \text{constant (constant energy per unit length of bridge)} \qquad (2)$$

and, consequently, additional firings at various current levels above the transition region should exhibit this characteristic, at least if appropriate allowance is made for random spread.

Percentiles in the all-fire region may be conveniently obtained by first firing two or three squibs at widely spaced currents above the transition region, and comparing the actual firing times with those suggested by the first-guess firing line.

If, as in the diagram that follows, these firings show a tendency toward a steeper slope than that of the template,
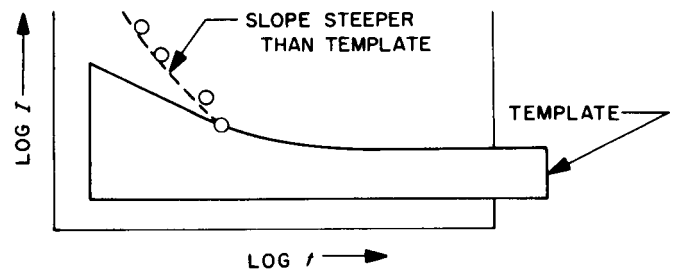
there may be a constant-time offset in the instrumentation; where the time-to-fire is variously taken as the time to onset of chemical reaction (as sensed by an electrostatic probe), the time-to-bridge-burnout, the time-to-shock (as measured by a vibration pickup), the time-to-rupture (as sensed by a pressure transducer), or the time-to-flash (as sensed by a photo-diode), at least the last three of these obviously represent times slightly in excess of the time-to-ignition.

This time offset should be minimized by recording the earliest measurable indication of ignition, and the residual offset should be corrected by tabulating $I^2(t - \alpha)$ for various (arbitrary) values of α and then selecting that value of α that gives the best fit.

Once the time-offset has been minimized, additional firings can be made in the all-fire region, making minor corrections to α if appropriate, and using the template as a guide to "expected" behavior.

The values of $I^2(t - \alpha)$ thus recorded will have a random spread; if, as is usually true, this spread can be reasonably approximated by a log-normal distribution, the mean and standard deviation of this distribution can be

obtained by plotting the results on probability paper (such as Keuffel and Esser 359-23), and drawing a straight line of best fit; percentiles can be calculated from

$$I^2 t = \bar{x}(1 + s)^{\pm TF} \qquad (3)$$

where $\bar{x}$ is the sample mean, s is the logarithmic standard deviation (both obtained from the probability plot), and TF is the tolerance factor (Ref. 4) appropriate to the number of samples and to the percentiles and confidence of interest. A six-sample example is given in Appendix B.

Should a straight line fail to give a reasonable fit on probability paper, examination of the deviations will usually reveal the cause, one of the most common being a poor estimate of the time offset α. With small sample tests, it is unlikely that any deviation from a true log-normal distribution can be detected; in Appendix C, three other common distributions (Weibull, Student's t, and Logistic) have been plotted on probability paper to illustrate how they tend to deviate from a normal distribution only at extreme percentiles.

D. Percentiles in the No-Fire Region; The "Ramp" No-Fire Test

In the no-fire region, Eq. (1) may be approximated by

$$I^2 = \text{constant (constant power per} \atop \text{unit length of bridge)} \qquad (4)$$

but, unlike the all-fire region where one variable (I) could be fixed and another (t) measured, some artifice must be used if we are to gain the small-sample advantages of tests by variables.

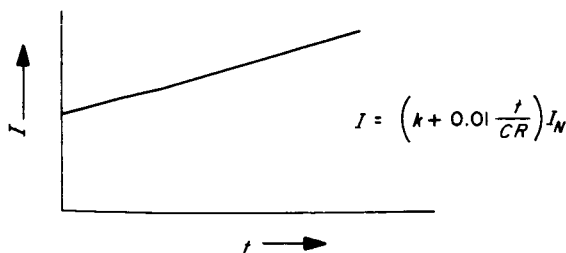The simplest method for doing this is to fire samples by applying a ramp current as in Fig. 4, where the initial

$$I = \left(k + 0.01\,\frac{t}{CR}\right)I_N$$

Fig. 4. Ramp current for tests in no-fire region

current is safely below the no-fire level indicated by the first-guess firing curve, and where the rate of current increase is of the order of 1% in each time interval corresponding to the thermal time constant.

The no-fire region corresponds to a region of thermal "balance"; by increasing the current in the fashion described, the current at which the squib fires will be only slightly in excess of the steady current needed (Appendix D).

For a squib with a 50% no-fire of 1 A and a thermal time constant of 20 ms, we might, for example, start at 0.5 A and increase the current at the rate of 0.01 A/20 ms (i.e., 0.5 A/s). Slower increases are preferable in the interest of accuracy, provided there is no reason to believe the firing characteristics are being changed by the consequent "preheating."

The firing currents of several squibs should be recorded, and the mean and standard deviation obtained as for those of $I^2 t$ in the all-fire region. If, as is likely, log I gives a good straight-line fit on probability paper, the percentiles may be calculated from

$$I = \bar{x}(1 + s)^{\pm TF} \qquad (5)$$

where the terms are comparable to those of Eq. (4).

It is possible that preheating may increase or decrease the squib no-fire levels; a check of this possibility may be made by testing some squibs at a fast rate of current rise, others at a slow rate, and comparing the two means and standard deviations.

E. Percentiles in the Transition Region

Although squibs subjected to currents of the order of the no-fire level tend to fire in the transition region rather than at comparatively long times, there seems to be no simple way to determine percentiles in this region.

Consideration of the electrothermal analogy of Appendix A leads to the conclusion that minor variations in squibs can produce three forms of random variation in firing characteristics (Fig. 5), and also to the conclusion that the template forms an appropriate envelope for each of these forms of spread.
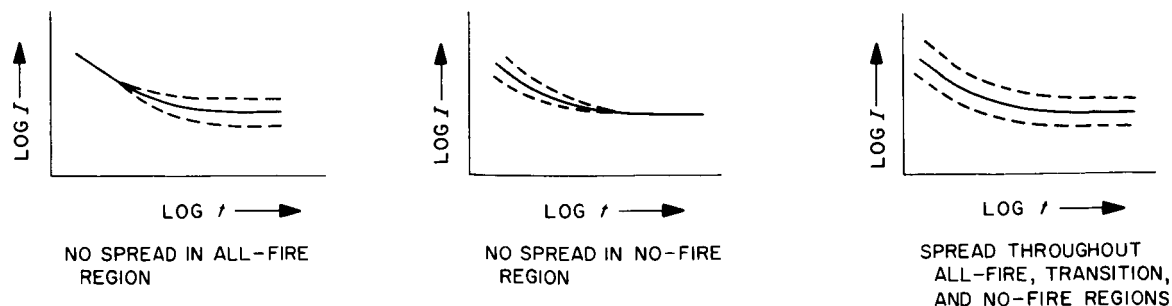
Fig. 5. Types of random spread

The same variations will produce variations in thermal time constants; as yet, no useful method has been devised for correlating the (measurable) random variations in thermal time constants with firing curve percentiles.

As firing times in the transition region obviously can be very sensitive to minor changes within the squibs, small-sample tests in the transition region (firing at fixed currents) can be useful in detecting minor changes attributable to environmental exposures or mechanical abuse.

The Ramp no-fire test minimizes a major weakness of the Bruceton no-fire tests that unavoidably and undesirably results in tests being made effectively in the transition rather than the no-fire region.

III. Predicting Firing Curves for Changes in Bridgewire Diameter

Once the template has been positioned for a particular squib, the firing curve for a similar squib differing only in having a bridgewire of another diameter can be predicted by shifting the template as indicated in Fig. 6 (Ref. 5).
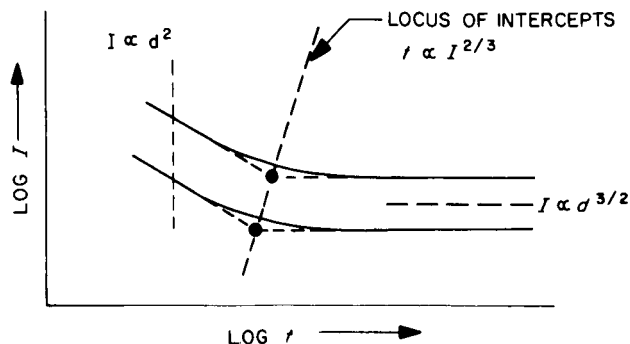
Fig. 6. Displacement of template appropriate to change in bridgewire diameter

IV. Squibs With More Than One Significant Thermal Time Constant

The small-sample test procedure as outlined is based on the premise that the squibs tested have only one significant thermal time constant; the universal template is appropriate only for such squibs. Unfortunately, some squibs do exhibit more than one significant time constant (Ref. 3) and must be handled as special cases (Appendix E).

Before using the procedure or the template indiscriminately, it is advisable to test fire one or two squibs at very high currents, and to get one or two long-time firings, as assurance that the template provides a good match for actual behavior; in particular, check for such an extended transition region as that in Fig. 7.
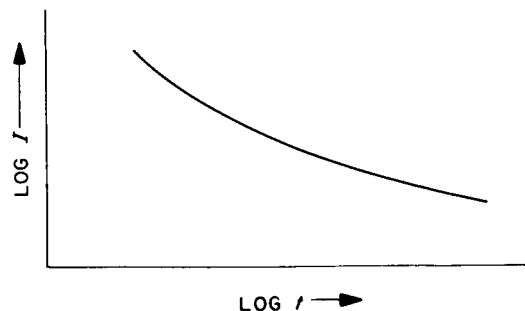
Fig. 7. Extended transition region typical of squibs having more than one significant thermal time constant

V. Number of Samples Required

Although some information can be obtained with only one test firing, a more realistic minimum would be about 18 samples, used as indicated in Fig. 8.

If fewer samples are available, modification would depend on the quantities of greatest interest; if more
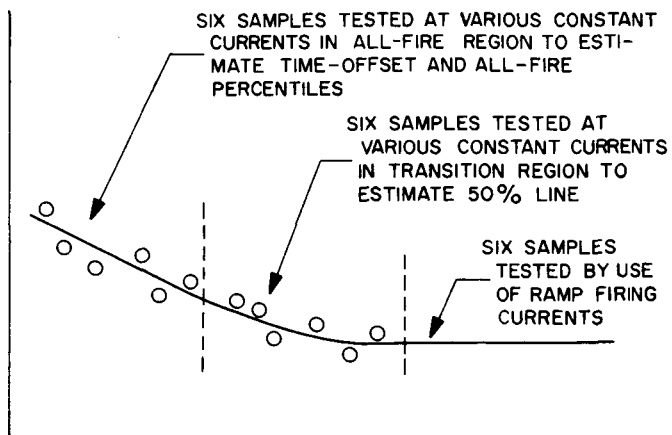
Fig. 8. The use of 18 samples to obtain information in each of the 3 regions

samples are available, the tolerance factors, and hence the spreads that have to be assumed for any given percentiles, could be reduced.

VI. Conclusions

The methods outlined in this report for squib firing time evaluation in the all-fire region have been in use at the Jet Propulsion Laboratory since 1962 without any anomalies having been detected. Both the universal firing template and the Ramp no-fire technique were introduced at JPL in 1965, and these, too, have manifested no anomalous characteristics. Both techniques have allowed detection of significant errors in Bruceton analyses as reported by other organizations.

Appendix A

Derivation of Equations Relating to Squibs With Only One Significant Thermal Time Constant

Squibs exhibiting only one significant thermal time constant may be represented by the highly simplified electrothermal analog[3] shown in the following diagram:
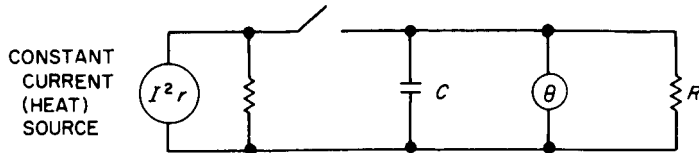
$I^2 r$ = HEAT (POWER) INPUT PER UNIT LENGTH OF BRIDGE (I = BRIDGE CURRENT, r = BRIDGE RESISTANCE PER UNIT LENGTH)

C = THERMAL CAPACITY OF BRIDGE PER UNIT LENGTH

R = THERMAL "RESISTANCE" OF BRIDGE TO BODY

θ = THERMAL POTENTIAL (TEMPERATURE) OF BRIDGE

The temperature of the bridgewire θ as a function of time t is given by the equation

$$\theta = I^2 r R (1 - e^{-t/CR}) \cdots \qquad \text{(A-1)}$$

Solving for the current required to bring the bridge to ignition temperature θ_i in time t,

$$I^2 = \frac{\theta_i}{rR} \times \frac{1}{1 - e^{-t/CR}} \qquad \text{(A-2)}$$

which may be expressed in terms of the current I_N corresponding to $t = \infty$

$$\left(\frac{I}{I_N}\right)^2 = \frac{1}{1 - e^{-t/CR}} \qquad \text{(A-3)}$$

Eq. (A-3) may be solved, in turn, for t:

$$t = CR \log \frac{1}{1 - \left(\frac{I_N}{I}\right)^2}$$

$$= CR\left[\left(\frac{I_N}{I}\right)^2 + \frac{1}{2}\left(\frac{I_N}{I}\right)^4 + \cdots + \frac{1}{n}\left(\frac{I_N}{I}\right)^{2n} + \cdots\right] \qquad \text{(A-4)}$$

from which it may be seen that in the all-fire region, where the ratios of I/I_N are large,

$$I^2 t \to CRI_N^2 \text{ (constant energy)} \qquad \text{(A-5)}$$

Similarly, Eq. (A-3) shows that in the no-fire region, where values of t/CR are large,

$$I^2 \to I_N^2 \text{ (constant power)}$$

or, more simply,

$$I \to I_N \qquad \text{(A-6)}$$

---

[3]Rather than derive the equations of this appendix by use of analogs, one could start equally well with the differential equation $C (d\theta/dt) + \theta/R = I^2 r t$ (Refs. 1, 2, and 3).

Appendix B

Analysis of All-Fire Results

For illustrative purposes, assume that the data in Table B-1 have been recorded for six samples.

Table B-1. Firing times and time constants

Sample	Time constant, ms	Current, A	Time, ms
1	19	3	4.84
2	22	4	3.30
3	13	5	1.73
4	24	6	1.88
5	18	8	0.97
6	16	10	0.81

Details of an analysis based on these results are given below; with practice, shortcuts will become self-evident.

I. Correction for Time Offset

A rough plot of the hypothetical test results in Table B-1 on log-log paper follows:
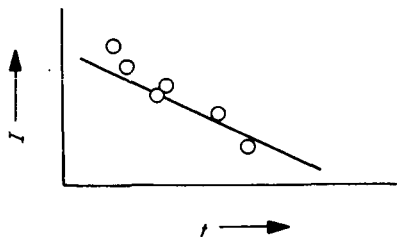
Because the curve of best fit is steeper than -0.5, a constant time offset is determined by assuming several values for α in $I^2(t - \alpha)$.

From Table B-2, the ratio of the maximum to minimum values of $I^2(t - \alpha)$ appears to reach a minimum in the vicinity of $\alpha = 0.2$–0.3; we will assume that $\alpha = 0.3$ is a reasonable choice.

II. Mean and Standard Deviation

Arranging the values of $I^2(t - 0.3)$ in an ascending order of magnitude, taking logs, and assigning percentage intervals to the magnitudes (assuming 7 intervals of 14.29% each) provides the data given in Table B-3.

Table B-3. Sequential arrangement of samples for probability plot

Sample	$I^2(t - 0.3)$	$\log_{10}$	Fraction of $(n + 1)$ samples embraced by magnitude, %
3	35.8	1.554	14.3
1	40.8	1.611	28.6
5	42.9	1.632	42.9
2	48.0	1.681	57.2
6	51.0	1.708	71.4
4	57.0	1.756	85.7

This data can be plotted on probability paper, and a straight line of best fit from which $\bar{x}$ and s are found (Fig. B-1) can be drawn.

Table B-2. Typical tabulation for determination of time offset

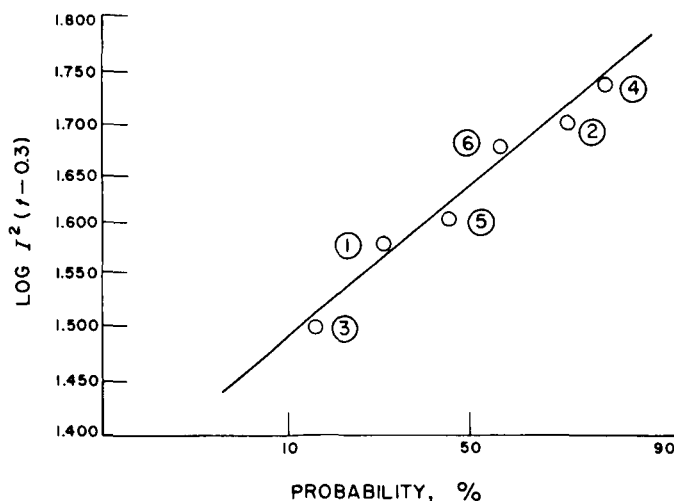| Sample	I^2	$I^2 t$	$I^2(t - 0.1)$	$I^2(t - 0.2)$	$I^2(t - 0.3)$	$I^2(t - 0.4)$	$I^2(t - 0.5)$
1	9	43.5	42.5	41.6	40.8	39.8	39.0
2	16	52.9	51.1	49.6	48.0	46.4	44.9
3	25	43.3	40.8	38.3	35.8	33.2	30.8
4	36	67.6	64.0	60.6	57.0	53.4	49.8
5	64	62.1	55.6	49.3	42.9	36.5	30.1
6	100	81.0	71.0	61.0	51.0	41.0	31.0
max		81.0	71.0	61.0	57.0	53.4	49.8
min		43.3	40.8	38.3	35.8	33.2	30.1
Ratio max/min		1.87	1.74	1.59	1.59	1.61	1.66

Fig. B-1. Probability plot

The linear plot of Fig. B-1 implies that $I^2(t-0.3)$ may be expressed in the form

$$I^2(t-0.3)_n = \bar{x}(1+s)^{\pm n} \tag{B-1}$$

where $\bar{x}$ is the mean, s is the (logarithmic) standard deviation, and n is the number of standard deviations above $(+)$ or below $(-)$ the mean.

Taking logs of Eq. (B-1),

$$\log[I^2(t-0.3)]_n = \log\bar{x} \pm n\log(1+s) \tag{B-2}$$

Log $\bar{x}$, read directly from Fig. B-1, equals 1.66. The value of s may be obtained by taking $n=1$ and reading $\log[I^2(t-0.3)]_n$, which equals 1.75, at the 84.1% level, which corresponds to one s above $\bar{x}$; we then have $\log(1+s) = 1.75 - 1.66 = 0.09.$[i]

Finally,

$$\bar{x} = \log^{-1} 1.66 = 45.8$$

and

$$s = \log^{-1}(0.09) - 1 = 0.23$$

─────────

[i]Log $(1+s)$ may be obtained more precisely by taking the slope over a greater range.

Note: As a check, s can be obtained by dividing the range $(1.756 - 1.554 = 0.202)$ by the constant 2.472 appropriate to six samples (Ref. 6); the result of 0.082 is in good agreement with our 0.09.

The firing times used in this example were deliberately selected to give a large s, and tolerance factors for reliabilities and confidence levels higher than those used would consequently lead to unrealistically wide bands.

from which

$$I^2(t-0.3) = 45.8(1+0.23)^{\pm n} \tag{B-3}$$

III. Calculation of Percentiles

From Ref. 4 we find that the two-sided tolerance factor for 99% reliability, 95% confidence, and six samples is 5.77; setting $n = 5.77$ in Eq. (B-3),

$$I^2(t-0.3) = 45.8(1+0.25)^{\pm 5.77}$$

$$= 45.8 \times \frac{1}{3.62} \text{ and } 45.8 \times 3.62$$

$$= 12.7 \text{ and } 166$$

For $I = 5\,A$,

$$t(1\%) = \frac{12.7}{25} + 0.3 = 0.51 + 0.3$$

$$t(50\%) = \frac{45.8}{25} + 0.3 = 1.83 + 0.3$$

$$t(99\%) = \frac{166}{25} + 0.3 = 6.6 + 0.3$$

IV. Drawing the Curve(s)

On the log I vs log t paper, lines can be drawn with slopes of -0.5 through $I = 5$ A and $t = 0.51$, 1.83, and 6.6 ms to give the 1%, 50%, and 99% firing lines.

No-fire levels can be *estimated* by aligning the template so that the template passes through each of the three points in turn, and so that the template intercept lies within the range of the measured time constants; the curves that result are shown in Fig. B-2.

The firing times shown in Fig. B-2 exclude the 0.3 ms correction; this correction must be added to any times read from the curves.

____ } CALCULATED PERCENTILES

—·— ESTIMATED PERCENTILES
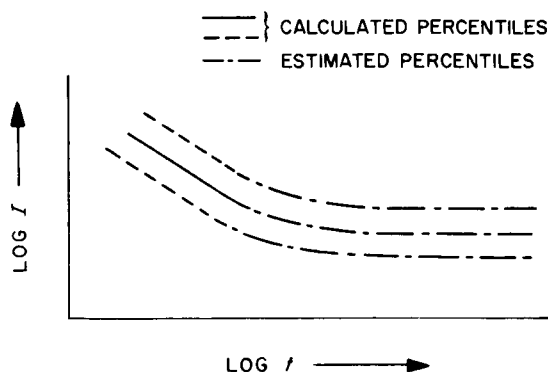
Fig. B-2. Calculated and estimated percentiles based on measurement of six thermal time constants plus six firings in the all-fire region

V. Use of a Computer

Because the analysis by hand of a large number of test results can be quite time consuming, the use of an electronic computer may prove economical.

A program for the IBM 1620, for example, has been prepared to make the following sequential steps:

(1) Calculate $I^2(t - \alpha)$ for a range of α in steps of 0.02 ms[5] from t_{min} to 0.40.

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[5]For some applications, it may be desirable to use steps smaller than 0.02 ms.

(2) Find that α which gives the smallest ratio of $I^2(t - \alpha)$ max to $I^2(t - \alpha)$ min.

(3) Calculate log $I^2(t - \alpha)$ for the α of step 2; print out.

(4) Find $\bar{x}_e$ and $s/\bar{x}_e$ for the values of $\log I^2(t - \alpha)$ calculated in step 3.

(5) Print out α, $\bar{x}(= \log^{-1} \bar{x}_e)$, and $s/\bar{x}_e$.

(6) Find $(t - \alpha)$ for a preselected value of I in $\bar{x}$.

(7) Find I for a preselected value of t in $\bar{x}$.

The computer approach differs from the manual calculations only in that s is obtained from

$$s = \left[\frac{\Sigma x_c^2}{(n-1)} - \frac{(\Sigma x_e)^2}{n(n-1)} \right]^{1/2}$$

rather than by a plot on probability paper or by use of the range (note, page 8).

As test results may sometimes contain systematic errors, computer calculations should be reviewed [possibly by plotting values of $\log_e I^2(t - \alpha)$ on probability paper] to minimize the chance of some such error being overlooked.
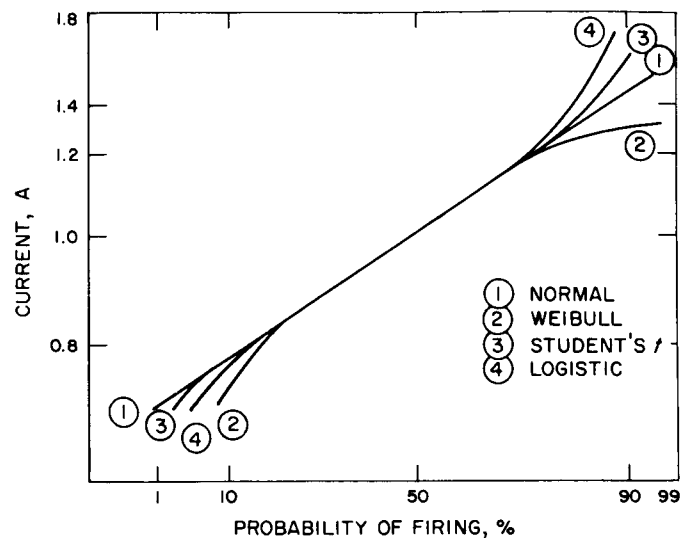
Appendix C
Comparison of Distributions

If simple manipulations with recorded values of I^2t [such as taking $I^2(t - \alpha)$] fail to produce a reasonably linear plot on arithmetic probability paper, it is possible that some form of distribution other than the log-normal is involved.

Figure C-1, however, illustrates that the differences between Weibull, Logistic, Normal, and t distributions may be so slight as to pass unnoticed if the sample size is small.[6]

If the distribution is *not* normal, estimates of percentiles cannot be made by the method detailed in Appendix B.

When a grossly nonlinear plot appears, test techniques and instrumentation should be reviewed critically; if no errors are apparent in these techniques, every effort should be made to evolve some rational method of normalizing the results.

[6]See Fig. 8, p. 23 of Ref. 7.

Fig. C-1. **Comparison of various distributions with parameters selected to give correspondence at the 35% and 50% points**

Appendix D

The Choice of Current Slope for the Ramp No-Fire Test

If the applied bridge current is of the form

$$I = (k + mt) I_N \tag{D-1}$$

and assuming, for simplicity, that the bridge resistance remains constant, the temperature θ is given by

$$\theta = I_N^2 R r \left\{ m^2 t^2 \left[1 - 2\frac{CR}{t} + 2\left(\frac{CR}{t}\right)^2 (1 - e^{-t/CR}) \right] + [k^2 + 2mk(t - CR)](1 - e^{-t/CR}) \right\} \tag{D-2}$$

or, in terms of the ignition temperature $\theta_i = I_N^2 R r$,

$$\frac{\theta}{\theta_i} = m^2 t^2 \left[1 - 2\frac{CR}{t} + 2\left(\frac{CR}{t}\right)^2 (1 - e^{-t/CR}) \right] + [k^2 + 2mk(t - CR)](1 - e^{-t/CR}) \tag{D-3}$$

For the extreme cases where $k = 0$, or $k = 1$ and $m = 0$, Eq. (D-3) reduces to fairly simple forms:

for $k = 0$,

$$\frac{\theta}{\theta_i} = m^2 t^2 \left[1 - 2\frac{CR}{t} + 2\left(\frac{CR}{t}\right)^2 (1 - e^{-t/CR}) \right] \tag{D-4}$$

and for $k = 1$,

$$\frac{\theta}{\theta_i} = (1 - e^{-t/CR}) \tag{D-5}$$

For cases where k is fractional, the situation is possibly best visualized by first considering the steady currents needed to fire the squib in a fixed time, as given by the equation

$$\frac{I}{I_N} = \left(\frac{1}{1 - e^{-t/CR}}\right)^{1/2} \tag{D-6}$$

Values of I/I_N for various values of t/CR (where t is the time to ignition) follow:

t/CR	I/I_N
1	1.26
2	1.07
3	1.03
4	1.01
5	1.003

It can be seen that a steady current less than 1% in excess of the no-fire current will fire the squib, provided that the current is sustained for $t > 4\,CR$.

If, instead of a steady current, a current of the form given below is applied, it follows that the current I_x, at a
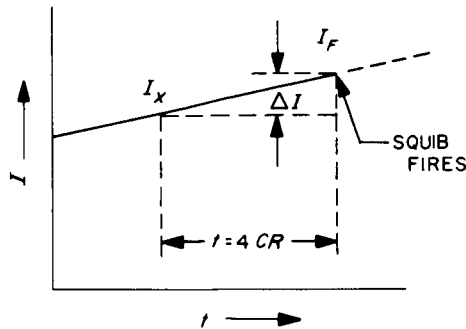
time $t = 4\,CR$ short of the firing time, does not exceed I_N by more than 1%, and, consequently, that I_F does not exceed I_N by more than 1% + ΔI.

Where the rate of increase is set at 0.01 I_N per $t = CR$, the gross error in I_F does not exceed $(0.01 + 4 \times 0.01)I_N$, or 5% of I_N; the actual error is somewhat less because the average power input in the interval between I_X and I_F is somewhat higher than that ruling for I_X.

The initial step of current has no significant effect on accuracy; it serves only to reduce test time and to minimize possible adverse effects of protracted heating.

The slower the rate of rise, the smaller will be the inherent error. If a rate of 0.001 I_N per $t = CR$ is chosen, and if the squib takes at least 5 units of time to fire, the above tabulation shows that $I_X/I_N \leqq 1.003$, and, with $\Delta I = 5 \times 0.001\,I_N$, the gross error should not exceed $(0.003 + 5 \times 0.001)\,I_N$, or 0.8%.

In practice, instrument accuracy may not allow current measurement to better than 1 or 2%, and the use of extremely slow rates of current increase may not be justified.[7]

It may be desirable to conduct tests at two or three distinctly different rates of rise to ensure, by comparing the results, that no significant second-order effects are present.

It is obvious that a non-linear slope could be used for the Ramp test; there may be circumstances where other such slopes offer advantages. When available test equipment allows only manual adjustment of the current, it may be convenient to test by stepping the current, holding each step for a time not less than about 4 CR.

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[7]The current ruling at the time of firing may be read from a panel meter, or, preferably, as displayed on a storage oscilloscope or clamp-and-hold digital meter.

Appendix E

Evaluation of Squibs With More Than One Significant Thermal Time Constant

The electro-thermal analog of a squib with one thermal time constant, as shown in the following diagram,
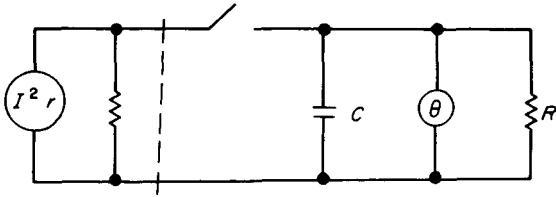
has a solution of the form

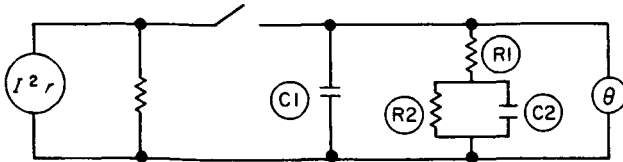$$\theta = A(1 - e^{-Bt}) \qquad \text{(E-1)}$$

where

$$A = I^2 r R$$

$$B = \frac{1}{CR}$$

as discussed in Appendix A.

For a squib with two significant thermal time constants such as the following electro-thermal analog,

the solution is of the form

$$\theta = I^2 r (n + p e^{-lt} + q e^{-mt}) \qquad \text{(E-2)}$$

where

$$n = \frac{D}{C}$$

$$l = \frac{B + E}{2A}$$

$$m = \frac{B - E}{2A}$$

$$p = \frac{A}{E}\left(mn - \frac{1}{C_1}\right)$$

$$q = \frac{A}{E}\left(\frac{1}{C_1} - ln\right)$$

and

$$A = R_1 R_2 C_2$$

$$B = C_1 + C_2 + \frac{R_1 C_1}{R_2}$$

$$C = \frac{1}{R_2}$$

$$D = \frac{R_1 + R_2}{R_2}$$

$$E = (B^2 - 4AC)^{1/2}$$

Thus, although the general form of Eq. (E-2) is simple, substitution of values for l, m, n, p, and q leads to a rather unpleasant form.

Algebraic solutions of Eq. (E-2) are a lot less straightforward than those for Eq. (E-1), and the presence of two time constants precludes preparation of a universal template.

If more than two significant time constants are present, the electro-thermal analog would obviously be even more difficult to handle.

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